Photometric and Spectroscopic Analysis for the Determination of Physical Parameters of an Eclipsing Binary Star System

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Introduction

Most of the stars seen in the night sky are members of a system of two or more stars. A binary star is a pair of stars that are bound together by gravity. A system of stars where one star passes in front of the other on a periodic basis is called an *eclipsing* binary. Eclipsing binary stars are so far away that they may only be observed from Earth as a single point of light and cannot be individually resolved. Eclipsing binary stars serve as the perfect vehicle for studying stellar dynamics due to the information that can be determined about them as a result of the physical properties that can be measured from the interaction of the two stars.
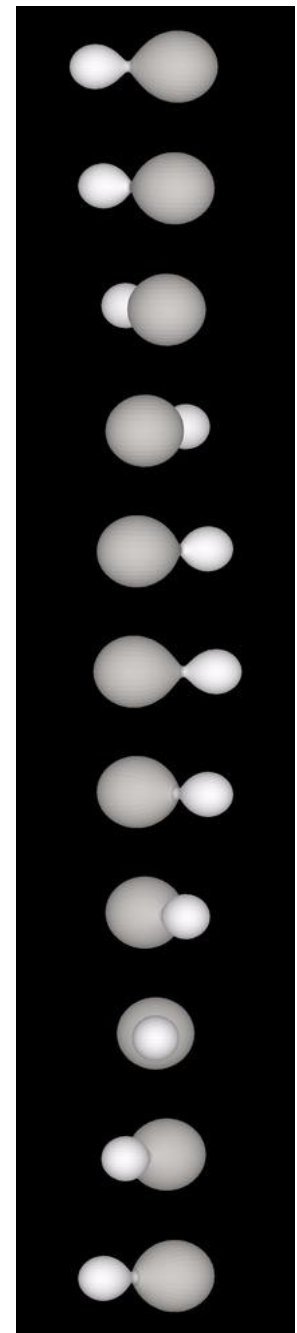
This research project addresses the question: can an amateur astronomer study this "pinpoint of light" and determine an array of physical data about the stars that make it up including rotational period, separation of the stars, radius, and mass of each star from her own backyard using a readily available telescope, tracking mount, CCD (charge coupled device) camera, and spectroscope?

Professional astronomers use the techniques of photometry and spectroscopy to study distant stars. Photometry is the measurement of the intensity of star light over a period of time. A light intensity curve versus time can be obtained with a telescope, mount and CCD camera. The BB Pegasi binary system is simulated in Figure 1 to show how the two stars rotate around each other. When the smaller star is behind the larger star (from earth's point of view) the absolute minimum in light intensity would be observed. When the larger star is behind the smaller star, the next minimum would be observed in a light intensity versus time curve. The time it takes for a complete cycle of rotation or rotational period can be calculated from measuring the distance from these two minima and multiplying by two. Spectroscopy of star light is performed by passing star light through a spectroscope and recording the image on a CCD camera. A spectroscope is an instrument that splits light into its component wavelengths. With a telescope, mount, spectroscope and CCD camera a plot of the wavelength components of a

star can be recorded and compared to data bases to determine its classification.

Figure 1 BB Pegasi Rotational Cycle Images generated with StarLightPro by Student. This shows the smaller star of the binary starting to the left (at the top of the image).

Materials

A Celestron C11 11 inch Schmidt Cassegrain type telescope was used for all data in this project. The light was then either directly imaged onto a CCD camera for brightness data or passed through a spectroscope first and then onto a CCD camera for intensity versus wavelength data. In order to do this type of work, the telescope must be on a special mount (in this case an Astrophysics 1200 mount) that will track the object under observation as time passes (due to the movement of the earth). The mount is computer controlled to track the object once the object has been located. The CCD camera or charge coupled device camera uses an array of pixels that converts light to electricity so the brighter the image, the more electricity produced. In this case, a QSI 583 WSG CCD camera was used which has an array of 3348 x 2474 pixels and each pixel is 5.4μm x 5.4μm. The spectroscope used for

this project was built by the student. It is an L200 Littrow Slit spectroscope. The spectroscope connects to the telescope and then the output of the spectroscope is recorded by the CCD camera when spectroscopy data is taken. The spectroscope uses a grating to spread the spectrum and allow analysis of the emission and absorption lines in the spectrum. These absorption and emission lines correspond to certain elements in a star (such as Hydrogen) which will allow the observer to measure the Doppler shift from the known locations of the lines in the case where a star is either moving to or away from the observer. The commonly known Balmer series corresponds to the absorption lines in the visible spectrum for Hydrogen. Since the exact wavelengths for absorption are known in the Balmer series, the change in wavelength due to the Doppler shift allows the astronomer to calculate how fast the star is moving with respect to the observer. The Doppler shift is the effect of light being shifted to a shorter wavelength (or more blue) when the light emitting object is moving towards the observer OR the effect of light being shifted to longer wavelengths (more red) when the light emitting object is moving away from the observer.

Once the data is acquired, software is used to assist in analyzing the data as discussed in the procedure below. This software includes: Maxim DL which is a software package that controls the QSI 583 camera and has image processing capabilities, RSpec which is for analyzing the spectrum and StarLightPro was used to simulate BB Pegasi.
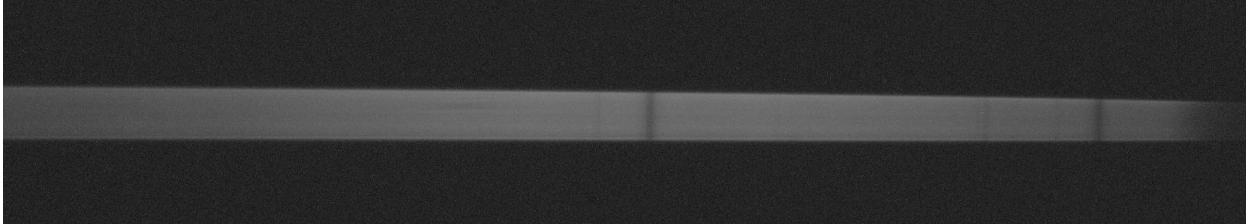
Procedure for Photometry (example using BB Pegasi)

The observation and data collection was done in the backyard of a suburban setting southwest of Austin, Tx. The telescope and other equipment were housed in a home observatory. The observatory roof was opened and the PC turned on which controlled the telescope mount using Sky6 software, the mount and CCD camera were also powered on. The Maxim DL software was started and used to collect data. TheSky6 software was used to slew (or move the telescope) to bright star Enif in the constellation Pegasus and take one second exposures with Maxim DL in order to adjust the telescope focus. Afterwards, the Find function in TheSky6 was used to slew the telescope to BB Pegasi and a reference star in this field of view was used to set the brightness. A reference star is a non-variable star (the brightness is stable) and is of known magnitude to be used as the night's brightness calibration star. This reference star sets the brightness scale and subtracts out variations in brightness due to effects in the atmosphere. Using Maxim DL a series of images of the reference star and BB Pegasi both in the field of view were collected, after increasing the exposure time to 8 seconds due to the fact that the reference star was fainter than BB Pegasi. In Maxim DL a sequence of 8 second exposures was set up and the auto guider was started to assure that alignment did not drift. The sequence was started at about 8pm and continued until about 2:30am at which time the hardware and software were shut down and the observatory roof was closed.

Procedure for Spectroscopy (example using 57 Cygni)

In order to begin collecting spectroscopic data it was first necessary to find the target eclipsing binary which in this case was 57 Cygni. The startup and focus were the same as for photometry discussed above; the exception being the spectroscope was mounted in front of the CCD camera. After focusing, TheSky6 software was used to slew the telescope to 57 Cygni. Next, the slit of the spectroscope was centered on 57 Cygni. At first it was necessary to open the slit wide in order to pan around and locate the star. Then narrow the slit to 30μm width to increase the resolution of the spectroscope. The spectroscope grating was adjusted to 17.7 millimeters to get a spectrum with the Hydrogen beta (Hβ)

absorption line centered. An example spectrum is shown in Figure 2. An exposure sequence was set up in Maxim DL, using 600 second (10 minute) exposures that collected data for 2 hours. This produced 12 images. The auto guider was on for tracking to ensure that the mount did not drift from its target. When the previous step was completed, Maxim DL was used to collect a series of dark and bias frames that were later used for calibration.


Figure 2 Raw spectrum of 57 Cygni

Analysis of Photometry Data (example using BB Pegasi)

Analysis of the photometric data was done using Maxim DL. The calibration process was first used to select the dark frames (a dark frame is an image taken with the telescope completely covered at the same exposure time as the light frames). Auto-Generate was run to average them and create a single master dark frame. Then the Batch Save and Convert command was used to select all of the image frames and perform dark calibration. After saving the calibrated images the script StackFitsImages was run. This script combined and stacked every eight images in a row out of the entire series to create a smaller series of images with much lower noise. All of these stacked images were opened and the Photometry popup was selected. With one image selected the aperture (circular cursor) was placed over BB Pegasi and it was selected as the new object. The reference star (in this case GSC 1682:1530) was selected. The magnitude of the reference star (in this case 11.5) was entered and the plot command was issued to produce a light curve or brightness versus time plot. This plot was used in calculating the period of orbit and the radius of the stars.

Analysis of Spectroscopy Data (example using 57 Cygni)

To begin analyzing the spectroscopic data MaximDL was opened and the calibration process was run. The folder containing the Dark frames was selected and Auto-Generate was used to average them and make a single master dark frame. Batch Save and Convert was used to calibrate all of the raw spectrum images from the series. Calibrated images were saved to a new folder. The Stack command was used to stack all 12 spectrum images to improve image quality. RSpec was started and the stacked spectrum image was opened and the aperture lines were used to select the spectrum band. The X-axis of the spectrum was calibrated using the readily identifiable Hydrogen Balmer absorption lines Hβ and Hγ as shown in Figure 3. The title, axis labels, and other identifiers were added in RSpec to complete the spectrum graph.
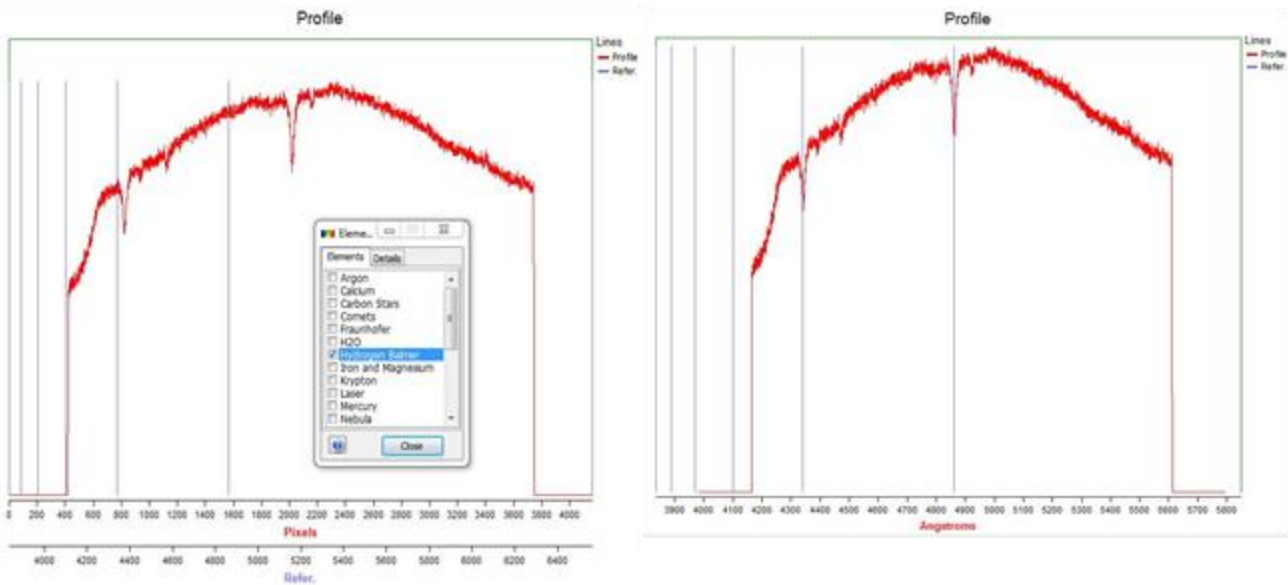
Figure 3 Screenshot from RSpec of x-axis calibration procedure.

Results

Period of Orbit

The period of orbit was computed from the light curve that was generated by photometry. A half period represents the time from one brightness minimum to the next successive brightness minimum because the minimum represent when the stars are in front of one another from the observers point of view. The lowest minimum is when the smaller star is completely eclipsed by the larger star. The next minimum is when the larger star is partially eclipsed by the smaller star. By measuring this time as shown in Figure 4, the brightness curve of BB Pegasi, a time for the full period can be computed.
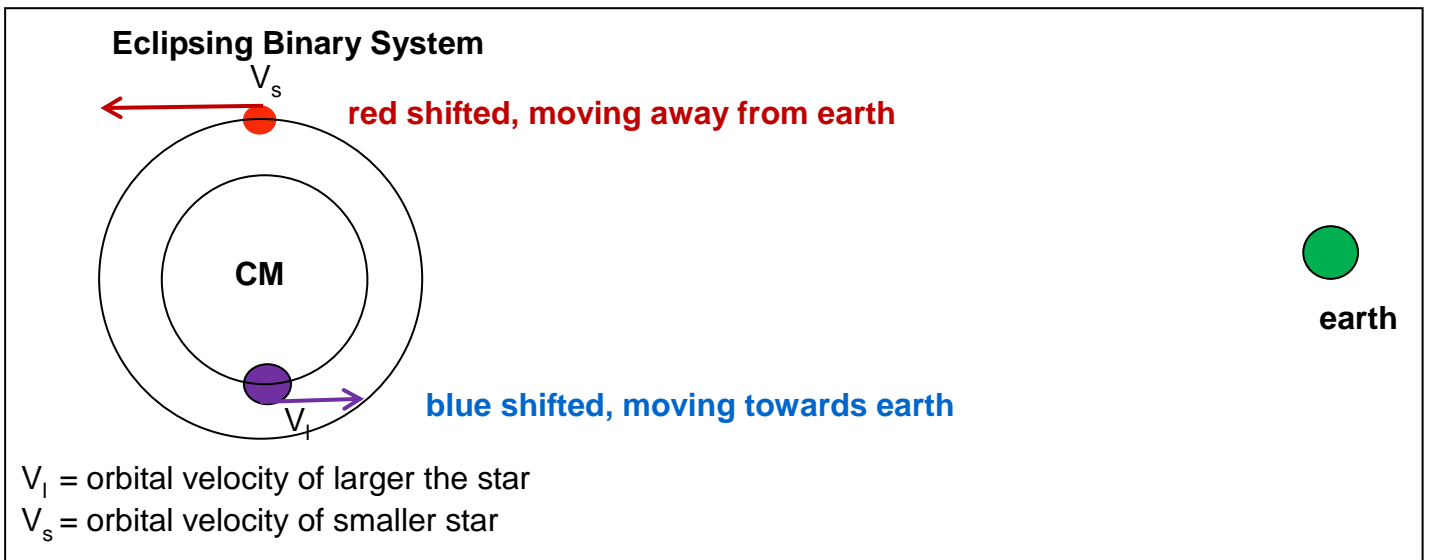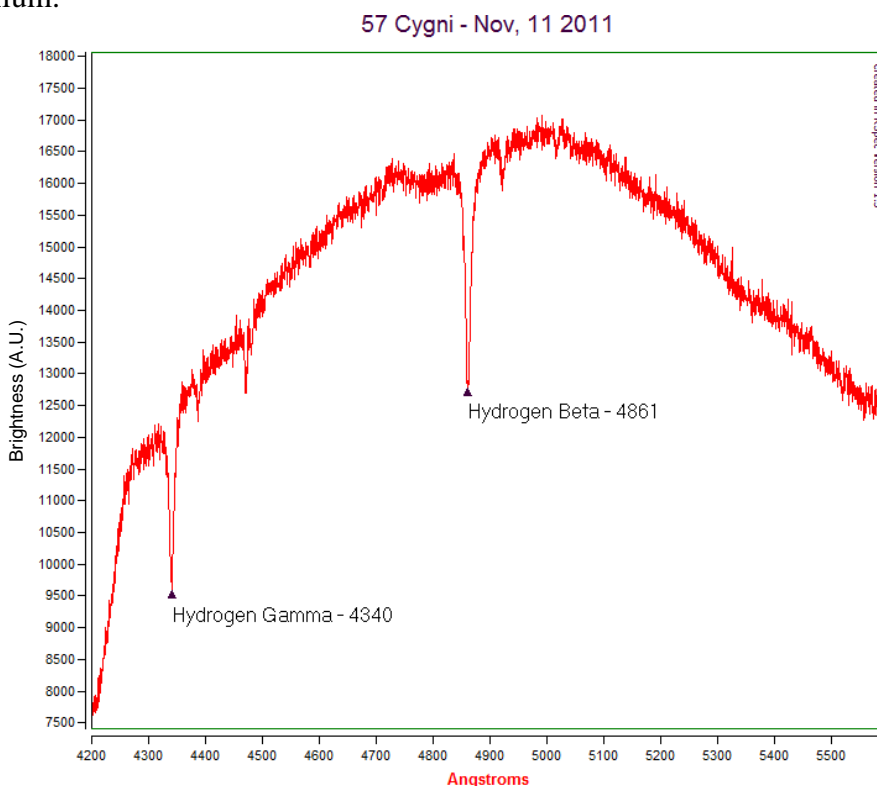
P = 8.74 hours = **31460 seconds**
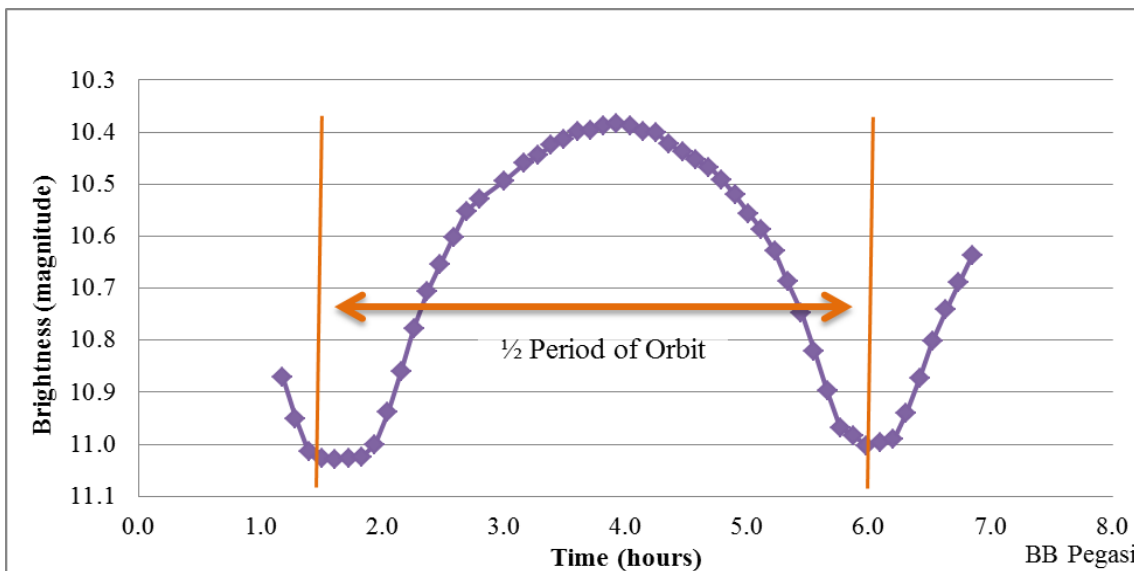
Figure 4 Brightness versus time graph for BB Pegasi using photometry.

Orbital Velocity

A binary star system has a common center of mass, CM, which is closer to the more massive star. The two stars orbit this center of mass, each with its own orbit. The larger star has a smaller orbit and moves slower than the smaller star. In the orbit, the orbital velocity of each star will pass a point when they are coming directly towards earth and that is when the orbital velocity is all "radial velocity" from earth's point of view- or velocity all directed to the observer on earth. The Doppler Shift is used to calculate the radial velocity (or velocity component of the orbital velocity that is directed to an observer on earth). When the Doppler shift is at a maximum to the observer on earth, then the star's orbital velocity is equal to the radial velocity as shown in Figure 5.


Eclipsing Binary System

V_s

red shifted, moving away from earth

CM

earth

blue shifted, moving towards earth

V_l

V_l = orbital velocity of larger the star
V_s = orbital velocity of smaller star

Figure 5 Two stars orbiting about a center of mass. The velocity of the larger star, V_l, is smaller than the velocity of the smaller star, V_s. When a star's velocity is all directed towards earth or opposite earth, the Doppler Shift will be a maxium.

57 Cygni - Nov, 11 2011

Created in RSpec Version 1.3

Brightness (A.U.)

Hydrogen Beta - 4861

Hydrogen Gamma - 4340

Angstroms

This orbital velocity can be computed by measuring the Doppler Shift of absorption line features of stellar spectra. The spectrum of 57 Cygni as shown in Figure 6 is centered on the Hydrogen Beta absorption line.

Figure 6 Spectrum of 57 Cygni.

In Figure 7, a narrower region of the spectrum is shown depicting data collected on the evening of November 11th which clearly indicates a single minimum at the 4861 angstrom Hydrogen Beta line. At this point in the orbit, there is no Doppler shift observed (there is a single minimum). This occurs because all the motion is perpendicular to the observer on earth. As the stars move from this point the Doppler shift gets larger and reaches a maximum when all the motion is toward or directly away from the observer on earth.
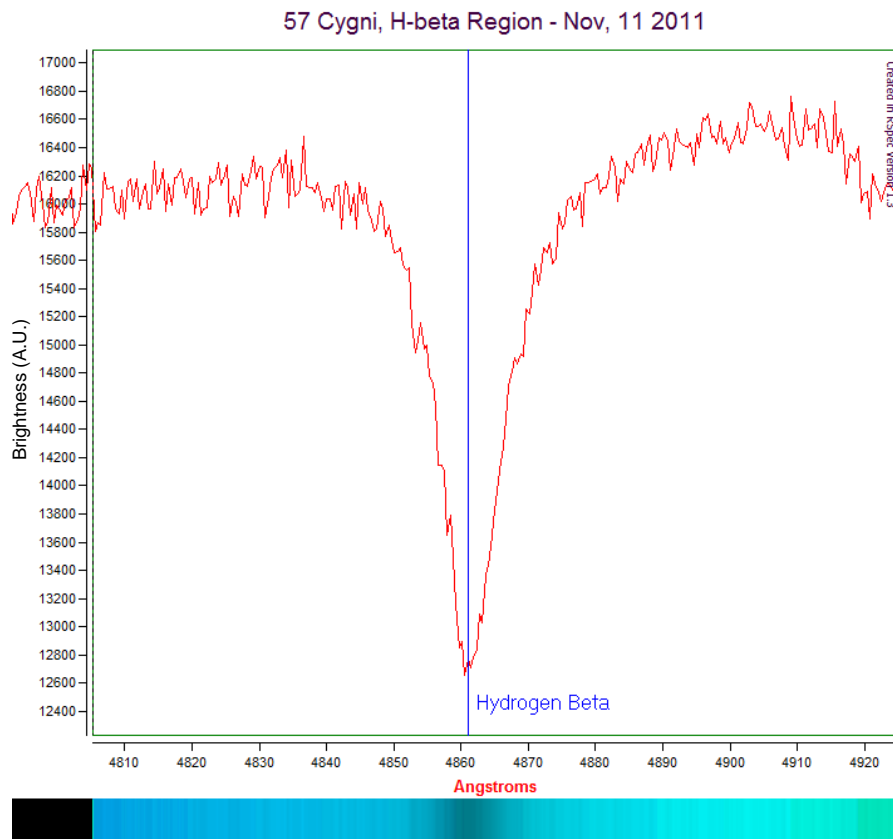

Figure 7 Narrow region of spectrum of 57 Cygni centered on the minimum at 4861Å hydrogen beta line. The spectrum was taken Nov, 11, 2011 and shows a single minimum.

The next spectrum was taken on the evening of November 16th, a spectrum of 57 Cygni was recorded which clearly indicates the Doppler Shift of the component stars as shown in Figure 8. The minimum of the blue shifted star which is moving towards the Earth is shifted to a shorter wavelength and the minimum of the red shifted star which is moving away from the Earth is shifted to a longer wavelength. Astronomers calculate orbital velocity by measuring the magnitude of these Doppler Shifts.
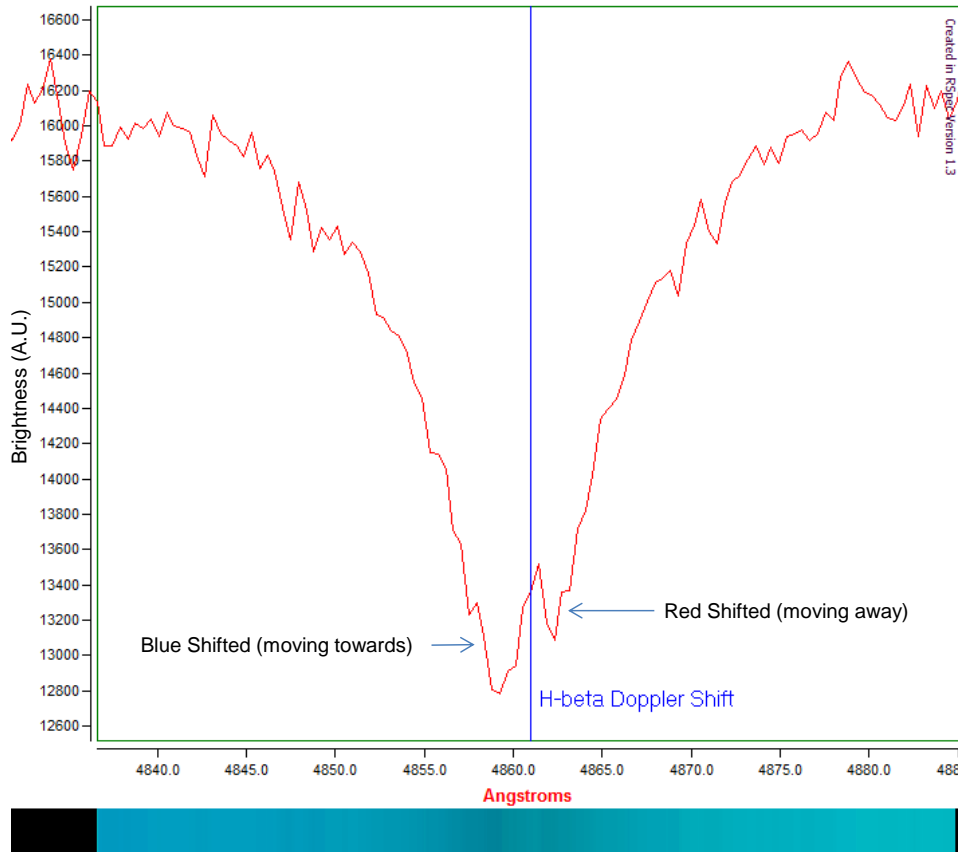
Figure 8 Spectrum taken Nov 16, 2011 showing Doppler shift for the two stars making the binary system of 57 Cygni. The blue shifted is to smaller wavelengths because this star is moving toward Earth. The red shifted is to longer wavelengths for the star moving away from Earth.

The orbital velocity of a star follows a simple relationship to the Doppler shift. The velocity is equal to the change in wavelength (Doppler shifted wavelength from single absorption line wavelength) divided by the single absorption line wavelength ratio then multiplied by the speed of light.

$$\Delta\lambda = \lambda_{new} - \lambda_{rest}$$

$$v = (\Delta\lambda/\lambda_{rest}) \times c$$

λ_{new} = new wavelength of absorption line, Doppler shifted

λ_{rest} = rest wavelength of absorption line, single absorption line

$\Delta\lambda$ = change in wavelength due to motion

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c = the speed of light, constant = 2.998×10^8 m/sec

v = orbital velocity of the star

Although many attempts were made to collect spectra of BB Pegasi it was determined that the faintness of this star (greater than 11th magnitude) made it impractical to obtain bright enough spectra for calculations. Even 30 minute spectra exposures did not reveal a bright enough spectrum to be usable. It was for this reason that the spectrum of the brighter 57 Cygni was used (above) to demonstrate measurement of rotational velocity Doppler Shift. In order to complete the rotational velocity calculations for BB Pegasi the data from Lu and Rucinski's *Radial Velocity Studies of Close Binary Star* was used to create the graph shown in Figure 9. By measuring the maximum velocities of the larger and smaller stars at both their blue shifted and red shifted phases and averaging the value for each star orbital velocities for each component were computed.

$V_1 \approx (65 + 136)/2 =$ **100 km/s (larger star velocity)**

$V_s \approx (291 + 226)/2 =$ **258 km/s (smaller star velocity)**

V_1 = velocity of larger the star

V_s = velocity of smaller star
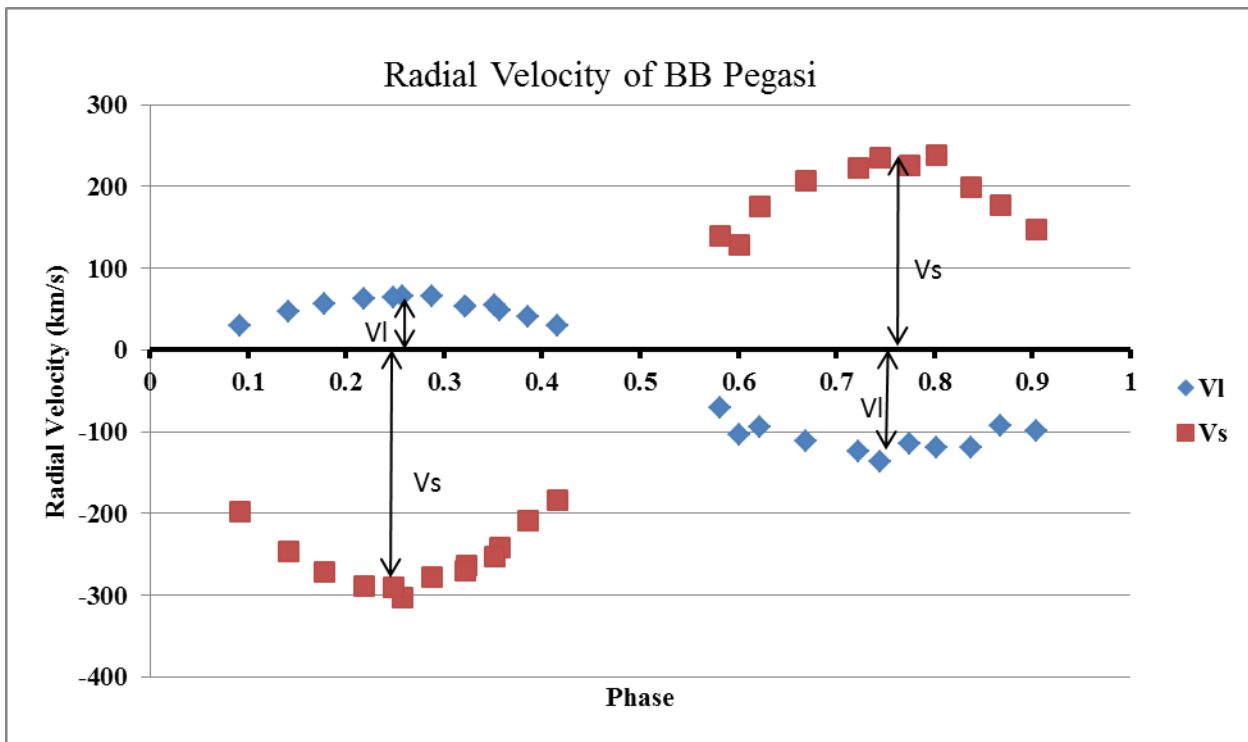
Figure 9 Radial velocities are shown for the binary BB Pegasi during several points as the two stars orbit each other. Data to do the calculation was taken from the reference Lu and Rucinski's due to the faintness of BB Pegasi. The blue data is the velocity, Vl, for the larger star which moves slower and the red data is the velocity, Vs, for the smaller star, which moves faster. The calculations were done using the same methods with Doppler shift as discussed. The radial velocities at the largest absolute values are averaged.

Radius of Each Star
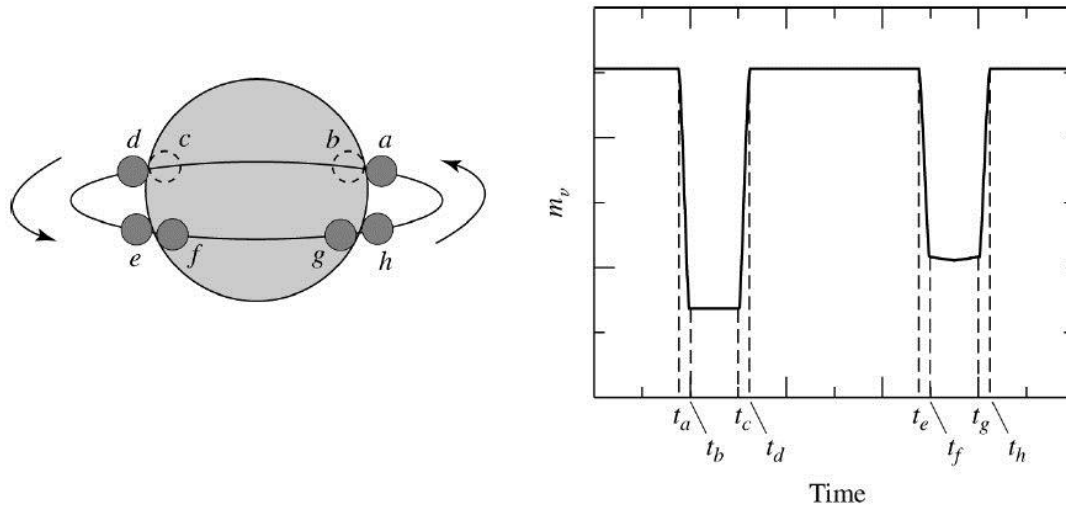
Figure 10 A diagram and representative light curve of intensity versus time for several points of interest in the orbits of the binary stars. At point A the small star is just going behind the large star and at point b it is just completely behind the large star, point C is just as it emerges and point D is when the small star is completely emerged, point E is as the large star is beginning to be behind the small star and point f is when it is completely behind the small star, point g is just as the small star emerges out from the front of the large star and point h is as the two stars are again just separated. The light curve to the right represents the effects of these key points on brightness. Diagram from Dasa Rozmus, Seminar November 2010.

Figure 10 shows a typical eclipsing binary light curve and a diagram of a smaller star and its larger companion orbiting one another. The light curve shows several regions that can be used to calculate the radius of the individual stars when you know the rotational velocities. The time from t_a to t_b represents the period when the smaller star is moving behind the larger star. This results in a decrease in the light intensity from the binary pair. From t_b to t_c the smaller star is complete hidden by the larger star and thus the light intensity is at a minimum. From t_c to t_d the smaller star is reappearing from behind the larger star and the light intensity is increasing again. By measuring the time that it takes for the smaller star to disappear or reappear and knowing the velocity (calculated above) it is possible to calculate the radius of the smaller star. Similarly, the time that the smaller star is hidden can be used to calculate the radius of the larger star.

This method is used for data collected for BB Pegasi from photometry done in the backyard observatory in Austin, Tx as shown below in Figure 11. The points t_b, t_c and t_d are shown on the graph and are used with the velocity calculated above to determine the radius of the two stars.
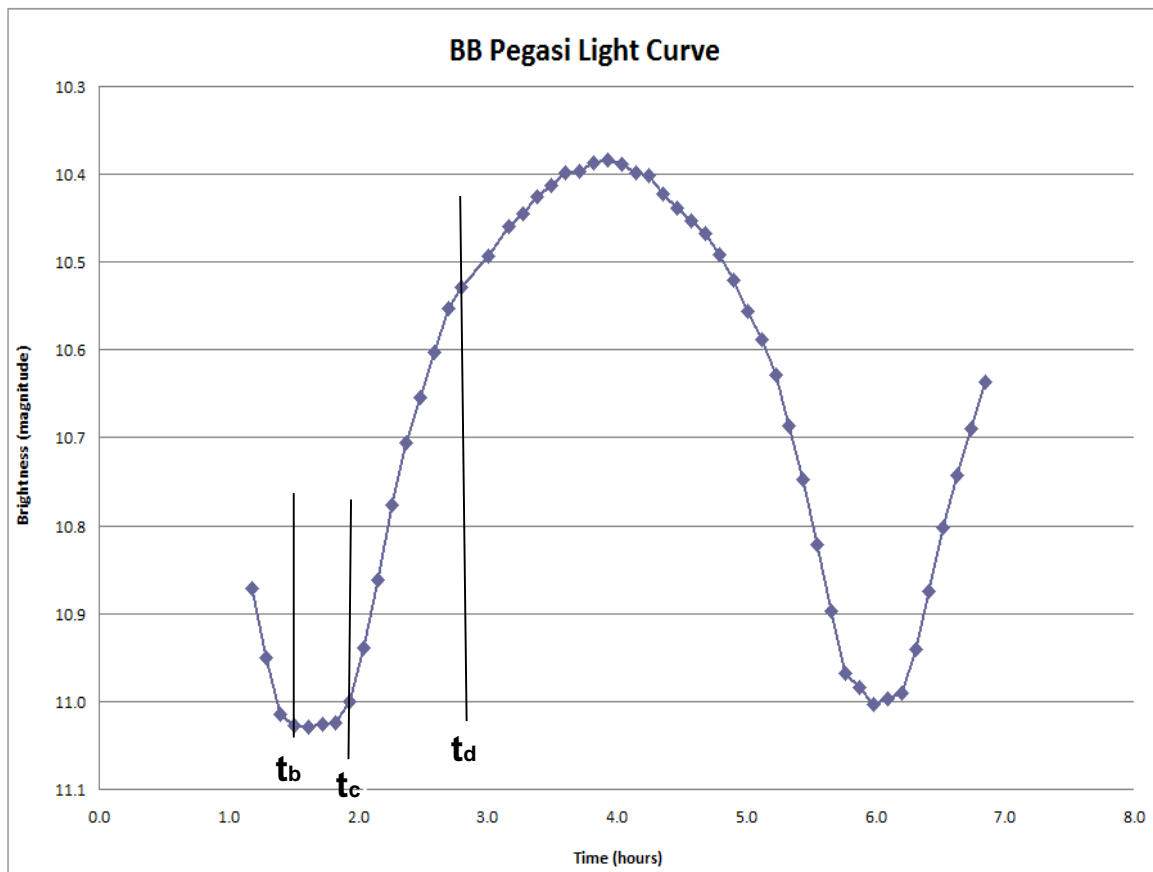
Figure 11 shows the light curve of BB Pegasi with t_b, t_c, and t_d identified. By measuring these times it was possible to calculate the radius of the smaller and larger stars.

The radius of the small star is calculated by using the radius of the small star as a distance travelled in a certain time at a known velocity. This is as determined by the time it takes from just starting to appear to full appearance (t_d-t_c) times the speed they are moving relative to each other, V which is V_s+V_l and then half of this is the radius.

$$r_s = (V/2)(t_d - t_c)$$

The large star radius is determined using the same method. The times are all found from the photometry data and the velocities were calculated from the Doppler shift as discussed earlier.

$$r_l = (V/2)(t_d - t_b)$$

where $V = V_s+V_l$

$t_b = 1.398$ hours $= 5032.8$ sec

$t_c = 1.830$ hours $= 6588.0$ sec

$t_d = 2.690$ hours $= 9684.0$ sec

$r_s = (358/2)(9684.0 - 6588.0) =$ **554184 km**

$r_l = (358/2)(9684.0 - 5032.8) =$ **832564 km**

$t_a =$ time when smaller star begins hiding behind larger star

$t_b =$ time when smaller star is completely hidden by larger star

$t_c =$ time when smaller star begins to reappear

$t_d =$ time when smaller star has completely reappeared

$V_s =$ velocity of small star

$V_l =$ velocity of large star

$r_s =$ radius of small star

$r_l =$ radius of large star

Separation Calculation

Each of the stars in the binary pair moves in an elliptical orbit. For small eccentricities (how squashed the ellipse is) the semi-major axis of each stars elliptical orbit can be estimated as a circle, see Figure 12. Since the rotational velocity of each of the stars is already known this can be used to estimate the semi-major axis (estimated to be the radius of a circle) of each star's orbit by multiplying the velocity by the orbital period and dividing by 2π. This is from the formula for the circumference of a circle where circumference is radius x 2π.

semi-major axis = (velocity * period) / 2π

$a_s =$ semi-major axis of small stars orbit
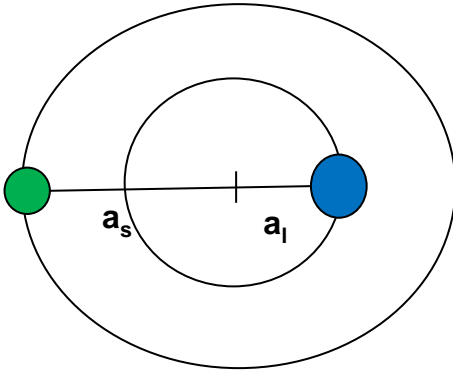
$a_l =$ semi-major axis of large stars orbit

$a_l = (V_l * P) / 2\pi = (100 * 31460) / (2 * 3.1415) = 500716$ km

$a_s = (V_s * P) / 2\pi = (258 * 31460) / (2 * 3.1415) = 1291847$ km

By summing the semi-major axis of both the smaller and larger stars orbits the separation of the two stars is calculated.

a = a$_l$ + a$_s$ = 1792564 km

Figure 12 Diagram of a binary system where a_s is the semi-major axis of the small star and a_l is the semi-major axis of the large star. This is the distance from the system center of mass to the small star or large star respectively.

Mass Calculations

Kepler's third law of planetary motion states that the square of the orbital period is directly proportional to the cube of the semi-major axis of the orbit. This law can be stated mathematically like so,

$$a^3/P^2 = G(m_l + m_s)/4\pi^2$$

where a is the separation between the two stars, P is the orbital period, m_l is the mass of larger star, m_s is the mass of smaller star, and G is the universal gravitational constant which is equal to 6.67384 x 10^{-11}.

The total mass (mass of smaller star plus mass of larger star) can be solved for by rearranging this equation.

$$m_s + m_l = 4\pi^2 a^3/P^2 G = (4)(3.1425)^2(1.7934 \times 10^9)^3/(31460^2 * 6.67384 \times 10^{-11}) = 3.447 \times 10^{30} kg$$

The ratio of the masses is known to be inversely proportional to the semi-major axis of the ellipses:

$$m_l/m_s = a_s/a_l = 1291847 / 500716 = \textbf{2.58}$$

This ratio can be used along with the total mass to compute the mass of each star.

$$2.58 * m_s + m_s = 3.447 \times 10^{30} kg$$

$$3.58 * m_s = 3.447 \times 10^{30} kg$$

$m_s = 9.628 \times 10^{29}$kg

$m_1 = 2.484 \times 10^{30}$kg

Using the value of 1.98892×10^{30} kg for the mass of our Sun we can express these star masses in terms of solar mass.

$m_s = 9.628 \times 10^{29}$kg = **0.48 solar mass**

$m_l = 2.484 \times 10^{30}$kg = **1.25 solar mass**

Results summary and comparison to professional calculations:

When compared to the professionally documented values for the physical parameters of BB Pegasi as presented in Lu and Rucinski's *Radial Velocity Studies of Close Binary Star* the numbers derived from the backyard measurements yield close results as shown in Table 1.

Parameter	Backyard	Professional	% Difference (Prof. vs. Backyard)
Rotational Period	8.739 hours	8.676 hours	0.7 %
Radius Large Star	832564 km	876330 km	5.0 %
Radius Small Star	554184 km	528580 km	4.8 %
Orbital Separation	1792564 km	1829165 km	2.0 %
Mass Large Star	1.25 solar mass	1.38 solar mass	9.4 %
Mass Small Star	0.48 solar mass	0.50 solar mass	1.0 %

Table 1 Comparison of calculated parameters using backyard photometry and spectroscopy to professional data published by Lu and Rucinski.

Conclusions

Amateur astronomical spectroscopy and photometry is very challenging and requires the astronomer to become proficient with many types of hardware and software. Telescope alignment and tracking are critical as is focusing and camera operation. The use of an astronomical spectroscope requires additional patience when engaged in the task of aligning a target star on the spectroscope slit. On top of these challenges the study of eclipsing binary stars adds the primary difficulty of locating a star with appropriate characteristics for study and bright enough to produce a spectrum that is not too faint to be

recorded. By careful use of consumer grade astronomical equipment it **is possible for a dedicated amateur astronomer to determine an array of physical parameters of a distant binary star system from a suburban setting** to a close approximation of professionally accepted values.

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